# A Path-Based Method for Feeder Partition Available Capability Evaluation under *N*-1 Security Criterion

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***Abstract*—A path-based algorithm is proposed in this paper to calculate feeder partition available capacity satisfying *N*-1 security criterion. Firstly, in order to simplify the analysis model and obtain more practical evaluation results, the distribution network is divided into feeder partitions according to automatic /manual isolated switch devices. Then, the topology of the distribution network is described by all load buses and the union of all power supply paths. After that, the evaluation of feeder partition available capacity is transformed into a series of mixed integer linear programming (MILP) problems. Finally, the effectiveness of the method is demonstrated by case studies from a practical 10kV feeder group distribution network in a provincial capital of China.**

***Index Terms*—Power supply path, *N*-1 security criterion, mixed integer linear programming (MILP), available capacity, feeder partition.**

## I. Introduction

As the rapid development of economy and the shortage of construction land, it is rather difficult to obtain the underground passage of feeders for supplying the new electricity demand. Thus, the evaluation of feeder capacity attracts lots of attentions recently[1]. In practice, the feeder available capacity can provide significant reference for the marketing department in distribution networks. However, it is obtained by artificial experience analysis traditionally, which is of low accuracy and efficiency. Considering the reliability of distribution systems, it would be critical to propose more accurate approaches for feeder available calculation.

There have been a few studies related to this issue. The load capability is estimated under the arbitrarily given load variations [2], without considering the load transfer under *N*-1 topology. A total supply capacity model is constructed based on feeder interconnection[3]. It considers feeder *N*-1 fault, but does not consider diversity of switch operations for load transfer. Practically, the layout of load and network sometimes needs to be changed to get maximum capacity.

Moreover, the judgment whether out-of-service load can be restored by the effective interconnection between feeders is significant when evaluating feeder *N*-1 available capacity. However, due to the large scale of distribution systems, the huge number of switching devices and the complicated and changeable modes of feeder interconnections would lead to great difficulties in analyzing load transfer. At present, there are mainly four kinds of load transfer algorithms, such as heuristic search algorithms [4], stochastic optimization algorithms [5], expert system methods [6], [7] and hybrid algorithms [8], [9].

This study introduces the concept of feeder partition[10] to not only simplify the analysis but also obtain more practical evaluation of available capacity. To provides meaningful data support to the power supply company, a novel mixed integer linear programming (MILP) algorithm is proposed to evaluate the feeder partition available capacity under *N*-1 security criterion. The results of this evaluation are more accurate and comprehensive for the presented method comprehensively considers the feeder interconnection and section synthetically.

The paper is organized as follows. Firstly, in Section II, using the feeder partition method, a path-based method is developed to describe the simplified network topology. Then, taking path states and the available capacity as variables, Section III formulates the *N*-1 security verification constraints and provides a MILP scheme for feeder partition *N*-1 available capacity calculation. Finally, test results from a 10kV feeder group distribution network in China show effectiveness and practicability of the proposed method, while Section V summarizes this paper.

## II. Path-Based Topology Description

### *A. Motivation*

The graph-based algorithm is widely used in topology analysis of electric networks. And the incidence matrix $\boldsymbol{A}$ is the simplest way to represent the relationship between buses and branches. However, the graph-based algorithm is not the best method for this issue. For one thing, some information of connected buses is omitted to assure linear independence of rows in $\boldsymbol{A}$. For another, when analyzing the transfer strategy after *N*-1 fault in distribution system, it is necessary to obtain not only connections between adjacent buses but also the radial structure of all buses. But the information above is rather difficult to obtain from procedures based on graph[11].

As well known, there are a large number of electricity customers in the distribution network. For simple analysis, with the characteristics of fault-spreading scope and power recovery range, the network can be divided into feeder partitions according to switch devices containing interconnection and sectional switches. Then the partition can be simplified as a load bus by means of joining all the load of electricity users in the zone. In this paper, buses are divided into load buses and substation buses, which refers to buses supplying power to load in the distribution

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network. Obviously, the branches and switch devices are corresponded one by one.

Ring construction and radial operation are typical characteristics of distribution networks. It is easy to know that, there are a set of alternative power supply paths for any load bus and only one of them is active when the distribution network is running. Then, the distribution network topology can be described by all load buses and all sets of power supply paths. Considering the above features, a path-based algorithm is proposed in this paper.

### *B. Path Definition*

Let $\pi_k^i$ be the $k$-th alternative power supply path associated to load node $i$, which is a set of branches connected to the substation bus.

If the number of paths associated to node $i$ is $p$, let the set $\Pi_N^i$ represent all of above paths:

$$\Pi_N^i = \{\pi_1^i, \pi_2^i, \ldots, \pi_k^i, \ldots, \pi_p^i\} \tag{1}$$

As noted above, to guarantee the radial operation, at most one of the paths in the set $\Pi_N^i$ is active. Thus, there comes a need to definite the binary variable $W_k^i$ to represent whether the path $\pi_k^i$ is active or not:

$$W_k^i = \begin{cases} 1, & \text{the state of path } \pi_k^i \text{ is active} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

[11] has been proved that the network is connected and radial if the following constraints are satisfied:

(1) Only one path in the set $\Pi_N^i$ is active, i.e.,

$$\sum_{\pi_k^i \in \Pi_N^i} W_k^i = 1, \quad \forall \text{load bus } i \tag{3}$$

(2) Any path $\pi_m^l$ contained in path $\pi_k^i$ must be active if path $\pi_k^i$ is active, which can be written as follows:

$$W_k^i \le W_m^l, \quad \forall \pi_m^l \subset \pi_k^i \tag{4}$$

Example system in Fig. 1 will be employed to illustrate the above definitions, where S1 and S2 are substation buses, A-C are load buses, and 1-7 are branches. Table I shows all alternative paths in the network.

If the example system is connected and radial, equalities (3) and inequalities (4) are both satisfied, that is:

$$\begin{cases} \sum_{\pi_k^A \in \Pi_N^A} W_k^A = 1 \\ \sum_{\pi_k^B \in \Pi_N^B} W_k^B = 1 \\ \sum_{\pi_k^C \in \Pi_N^C} W_k^C = 1 \end{cases} \tag{5}$$

$$\begin{aligned} & W_2^B \le W_1^A, \quad W_1^C \le W_1^A, \quad W_2^A \le W_1^B \\ & W_4^B \le W_3^A \le W_2^C, \quad W_3^C \le W_4^A \le W_3^B \end{aligned} \tag{6}$$

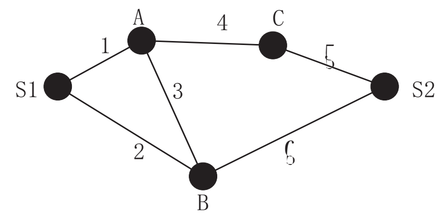

Fig. 1 Simple distribution network topology

TABLE I LOAD NODE PATHS FOR EXAMPLE SYSTEM

| nodes | Paths | Path nodes | Path branches |
|---|---|---|---|
| A | $\pi_1^A$ | S1,A | 1 |
| | $\pi_2^A$ | S1,B,A | 2,3 |
| | $\pi_3^A$ | S2,C,A | 4,5 |
| | $\pi_4^A$ | S2,B,A | 3,6 |
| B | $\pi_1^B$ | S1,B | 2 |
| | $\pi_2^B$ | S1,A,B | 1,3 |
| | $\pi_3^B$ | S2,B | 6 |
| | $\pi_4^B$ | S2,C,A,B | 3,4,5 |
| C | $\pi_1^C$ | S1,A,C | 1,4 |
| | $\pi_2^C$ | S2,C | 5 |
| | $\pi_3^C$ | S2,B,A,C | 3,4,6 |

The depth-first search is employed in this paper to obtain all of the candidate paths. Besides, in order to formulate the other electrical constraints, the second and the third sets of paths are defined as

$$\Pi_B^i = \{\text{Set of paths containing branch } b_i\},$$

$$\Pi_S^i = \{\text{Set of paths associated to substation bus } S_i\}.$$

## III. MILP BASED FEEDER AVAILABLE CAPACITY EVALUATION MODEL

### *A. Feeder N-1 Security Verification*

Feeder $N$-1 security criterion requires that fault isolation and power recovery can be achieved when an arbitrary component in feeders breaks down. According to feeder connections, load in power loss area can be transferred to other feeders by changing some states of switch devices. It means that there must be at least one operation mode to guarantee the reconfiguration network can run radially. Thus, after an arbitrary fault in feeders, besides constraints (3)-(4), the reconfiguration network should be satisfied with follows:

$$\sum_{\pi_k^i \in \Pi_N^i \cap \Pi_S^j} W_k^i \cdot L_i \le S_j \tag{7}$$

$$\sum_{\pi_k^i \in \Pi_N^i \cap \Pi_B^m} W_k^i \cdot L_i \le B_m \tag{8}$$

where $L_i$ = installed capacity of node bus $i$; $S_j$ = capacity of substation node $j$; $B_m$ = maximum ampacity of branch $m$.

The variables of constraints above are all states of load supply paths. Inequalities (7) are power supply capacity constraints,

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which require that the sum power of loads supplied by substance node $j$ is no more than its capacity. Feeder power flow constraints (8) indicate that the load flow of the branch $m$ is no more than its maximum ampacity.

If there exists at least one set of path states satisfying constraints (3)-(4) and (7)-(8), the load transfer will be achieved. Then the operation of switch devices can be obtained by path states. Otherwise, the analysis of load transfer is failed and the network is dissatisfied with $N$-1 security criterion.

The study focuses on the overload of components, which is the same as the treatments mentioned in the current literature when calculating the power supply capability. Thus, there is some simplification for the analysis of reactive power, bus voltage and network loss. Because feeders in the city distribution network are usually short and local reactive power compensation is widely used to adjust node voltage, the voltage drop is small and even can be ignored. What's more, the network loss is approximately included in the capacity of the load node.

### *B. Available Capacity Calculation Model*

The ranges of power supply sections are generally clear, and do not overlap and interweave in normal operation. Considering the large scale of the distribution network, the study targets feeder groups. There are numbers of components in the feeder group. If considering all component faults, the problem will be extremely complex. Thus, in the procedure of feeder $N$-1 security verification, only export line faults, which are most seriously in all feeder faults, are taken into consideration. Besides, the maximum load of the whole year is utilized to calculate the available capacity, which is commonly accepted in distribution system planning.

If the group feeder network satisfies the $N$-1 security criterion, there must be an available capacity when no fault or an arbitrary export line fault occurs. Supposing the number of feeders in the feeder group is $\alpha$, the $N$-1 available capacity of feeder partition is the minimum value of all $\alpha$+1 calculation results.

In this section, a novel MILP model will be presented to calculate the feeder available capacity in any condition mentioned above. The objective function and constraints of the available capacity can be expressed as

$$\max C_l^\beta \tag{9}$$

such that

equalities (3) and inequalities (4)

$$\sum_{\pi_k^i \in \Pi_N^i \cap \Pi_S^j} W_k^i \cdot L_i + \sum_{\pi_k^l \in \Pi_N^l \cap \Pi_S^j} W_k^l \cdot C_l^\beta \le S_j \tag{10}$$

$$\sum_{\pi_k^i \in \Pi_N^i \cap \Pi_B^m} W_k^i \cdot L_i + \sum_{\pi_k^l \in \Pi_N^l \cap \Pi_B^m} W_k^l \cdot C_l^\beta \le B_m \tag{11}$$

Where $\beta$=0,1,2,$\cdots$, $\alpha$ , $C_l^\beta$ = available capacity of load node $i$ in case $\beta$.

The same as inequalities (7)-(8), inequalities (10) are power supply capacity constraints and inequalities (11) are feeder power flow constraints. The variables contain not only path states but also available capacity, so sums of two variables multiplication $W_k^l \cdot C_l^\beta$ appeared in constraints (10)-(11), resulting in severe calculation difficulties. By introducing variables $Z_k^{\beta_l}$, the programming problem can be easily coped with.

$$Z_k^{\beta_l} = W_k^l \cdot C_l^\beta \tag{12}$$

It can be proved that the following inequalities (13)-(14) are equivalent to equality (12).

$$\left| Z_k^{\beta_l} \right| \le M \cdot W_k^l \tag{13}$$

$$\left| Z_k^{\beta_l} - C_l^\beta \right| \le M \cdot \left(1 - W_k^l\right) \tag{14}$$

where M is a large enough constant.

Taking into account $W_k^l = 0$ or $W_k^l = 1$, inequalities (13)-(14) can be rewritten as

$$\begin{cases} W_k^l = 0 \\ Z_k^{\beta_l} = 0 \\ \left| C_l^\beta \right| \le M \end{cases} or \begin{cases} W_k^l = 1 \\ Z_k^{\beta_l} \le M \\ Z_k^{\beta_l} = C_l^\beta \end{cases} \tag{15}$$

Thus, equalities (12) can be obtained.

On the basis of the equivalence relation above, constraints (10)-(11) can be transformed into the following expressions

$$\sum_{\pi_k^i \in \Pi_N^i \cap \Pi_S^j} W_k^i \cdot L_i + \sum_{\pi_k^l \in \Pi_N^l \cap \Pi_S^j} Z_k^{\beta_l} \le S_j \tag{16}$$

$$\sum_{\pi_k^i \in \Pi_N^i \cap \Pi_B^m} W_k^i \cdot L_i + \sum_{\pi_k^l \in \Pi_N^l \cap \Pi_B^m} Z_k^{\beta_l} \le B_m \tag{17}$$

which are linear and easy to be solved.

To sum up, taking the introduced variables, path states and available capacity as variables, the MILP problem consists of objective function (9), subject to constraints(3),(4),(16)and(17).

### *C. Optimization Framework*

The estimated available capacity feeder partitions can provide meaningful support to the power supply company. The $N$-1 available capacity of feeder partition $l$ in the network can be calculated via the following steps:

a) According to switch devices, divide the network into feeder partitions and sum up all loads in the feeder partition.
b) Search paths to describe the network.
c) Construct and solve the MILP model according to (3), (4), (9), (16) and(17) on the condition of no fault to obtain the capacity $C_l^0$.
d) Set $\beta$=1.
e) Suppose a fault occurs in the $i$-th feeder export, search paths and calculate the capacity $C_l^\beta$ by the same method as step b) and c). If there is no feasible solution, exit the calculation and output <not satisfied with $N$-1 security criterion>. Otherwise, continue the next step.
f) If $\beta < \alpha$, $\beta$= $\beta$+1, take step e); Otherwise, jump out the cycle, $C_l = \min\left\{C_l^0, C_l^1, C_l^2, \cdots, C_l^\alpha\right\}$.
g) Output results.

## IV. Case Study

### *A. Overview of the Test System*

In this section, the proposed algorithm is tested on a real feeder group from a distribution network of a provincial capital in China. By means of dividing the feeder group network into

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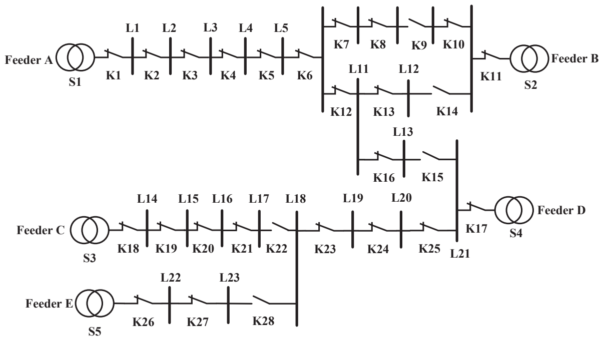


Fig. 2 Example network

feeder partitions according to the method discussed in Section II, simplified network with 5 feeders, 23 feeder partitions and 28 switch devices containing 5 loop switches and 23 sectionalizing switches, can be obtained and shown in Fig. 2. The algorithm is coded in the IntelliJ IDEA environment on a laptop equipped with Intel(R) Core(TM) i5-3337U CPU @1.80GHz and 4.0GB of RAM memory.

### *B. Numerical Study and Analysis*

Before the calculation, there is a need to search sets of alternative power supply paths of load buses, sets of node paths sharing any branch and sets of node paths associated to any substation bus using the method discussed in Section II. Table II lists the installed capacity and the path number of all feeder partitions.

After path searching, there are 187 alternative power supply paths of all load nodes in the example distribution system. The numbers of paths associated to the substation S1-S5 are 40, 45, 34, 34 and 34 respectively. Paths sharing branch K16 are up to 113, which accounts for 60.43% of all paths, illustrating that branch K16 is the most critical branch. Then, the entire network topology presented in Fig. 2 can be described as the union of all node paths above. And it is also helpful to formulate the electrical constraints.

Table II INSTALLED CAPACITY AND PATH NUMBER OF FEEDER PARTITION

| Feeder partition | Installed capacity/MVA | Path number | Feeder partition | Installed capacity/MVA | Path number |
|---|---|---|---|---|---|
| L1 | 2.3 | 9 | L13 | 0.63 | 7 |
| L2 | 1.33 | 9 | L14 | 2.52 | 7 |
| L3 | 0.63 | 9 | L15 | 1.26 | 7 |
| L4 | 4.8 | 9 | L16 | 0.76 | 7 |
| L5 | 0.16 | 9 | L17 | 0.4 | 7 |
| L6 | 2.825 | 9 | L18 | 0.65 | 7 |
| L7 | 0.21 | 10 | L19 | 1.8 | 7 |
| L8 | 2.52 | 10 | L20 | 0.645 | 7 |
| L9 | 0 | 10 | L21 | 1.13 | 7 |
| L10 | 4.8 | 9 | L22 | 13.47 | 7 |
| L11 | 0 | 7 | L23 | 0.88 | 7 |
| L12 | 1.6 | 10 | | | |

Table III AVAILABLE CAPACITY OF FEEDER PARTITION UNDER *N*-1 SECURITY CRITERION

| Feeder partition | Available capacity/MVA | time/s | Feeder partition | Available capacity/MVA | time/s |
|---|---|---|---|---|---|
| L1 | 5.529 | 3.979 | L13 | 5.529 | 4.003 |
| L2 | 5.529 | 2.556 | L14 | 10.169 | 1.516 |
| L3 | 5.529 | 3.841 | L15 | 10.169 | 2.116 |
| L4 | 5.529 | 11.58 | L16 | 10.169 | 1.790 |
| L5 | 5.529 | 3.668 | L17 | 10.169 | 1.921 |
| L6 | 5.529 | 7.854 | L18 | 0.759 | 2.055 |
| L7 | 8.444 | 1.742 | L19 | 0.759 | 1.632 |
| L8 | 8.654 | 2.715 | L20 | 0.759 | 1.671 |
| L9 | 11.174 | 1.937 | L21 | 0.759 | 1.385 |
| L10 | 11.174 | 3.527 | L22 | 0.759 | 7.396 |
| L11 | 5.529 | 1.565 | L23 | 0.759 | 1.235 |
| L12 | 11.174 | 2.308 | | | |

Under the assumptions of conditions of no fault and any export feeder fault, the MILP models are formulated evaluate feeder partition *N*-1 available capacity. By calling the open-source mixed integer programming solver Coin-or branch and cut (Cbc) through Java Native Interface (JNI), the branch and bound method is used to solve the MILP models. If there is no solution when one or several feeders are out of action, it means that the example system is not satisfied with *N*-1 security criterion.

Through the simulation, the example network can pass *N*-1 security verification. The computing time and the results of feeder partition *N*-1 available capacity are shown in Table III.

As we can see, there are many alternative paths, causing lots of difficulties in analyzing the load transfer scheme. If the genetic algorithm is employed to accomplish the network reconfiguration after feeder fault, it would cost lots of time for the crossover and mutation of genetic operators. In this paper, the issue above is transformed into the MILP models considering all of sectionalizing and loop switches in the network, with higher computation accuracy and efficiency compared with heuristic search algorithms and stochastic optimization algorithms.

Comprehensively analyzing Fig. 2, Table II and Table III, it comes to the conclusion that the available capacity of a feeder partition is influenced by not only the installed capacity of itself but also the installed capacity of the adjacent partition and the interconnection of feeders. For instance, the installed capacity of feeder partition L22 is up to 13.47MVA. If feeder E is out of order, the load transfer capacity is large. Considering the interconnection of feeders and the capacity installed in each feeder, load in power loss area can only be transferred to feeder D. Therefore, the available capacity of feeder D and E is fairly small, that is to say, the available capacity of feeder partition L18-L23 is low to 0.759MVA. For feeder partition L9, L10 and L12, there are quite many connected feeders and they are under low load. Thus, the available capacity of them is up to 11.174MVA.

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## V. Conclusions

In this paper, a novel path-based algorithm is proposed to evaluate the available capacity of feeder partitions under *N*-1 security criterion. Comparing with the export system methods which need to establish a knowledge base, the method based on the network topology can cover a larger range. And the evaluation results of the available are more accurate and easier to calculate by solving the proposed MILP models.

What's more, by changing the objective functions, the path-based reconfiguration method of distribution network has wide applications. For example, if the load transfer scheme is subject to the minimum number of switch actions, the 0-1 linear programming model would be constructed after employing path states to describe the states of all switch devices.

With the fast development of the distribution network, power supply reliability has received extensive attention. Considering the practical marketing demand of the power supply company together, the method proposed in this paper is of good use and significant application prospect. In the further study, the author plan to concentrate on the network reconfiguration scheme considering voltage constraints during the operation of distribution systems.

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